| Year 3 - addition |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |  |  |  |
| To add numbers with up to three digits, using formal written methods of columnar addition numbers up to 1,000. | $38+23=$ <br> Use straws alongside the calculation to add. Children encouraged to use 'bunches' of ten when possible. This should progress onto using base 10 alongside the calculation when numbers get larger. Children should be taught when to exchange while | Draw place value counters in a place value chart / be presented with pictures of place value counters in a place value chart to add. Children should be taught when to exchange - relating to their work in the concrete and be able to show this in their drawings. <br> (Showing the abstract alongside the pictorial can help children make the connection). | $\begin{array}{r} 369 \quad+\begin{array}{c} 300+60+9 \\ +148 \\ \hline \end{array}+100+60+8 \\ \hline 400+100+17=517 \end{array}$ <br> Start the formal written method by partitioning the numbers into clear place value columns (relating to the concrete and pictorial work). Children should be taught when to exchange and be able to show this in their calculations. |  |  |  |



| KEY <br> VOCABULARY | digits | addition | add | more sum value column | altogether exchange | total regroup | columnn method | place |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| Year 3 - subtraction |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To subtract numbers with up to three digits, using formal written methods of columnar subtraction numbers up to 1,000. | Use base 10 alongside the calculation to subtract. Children should be taught when to exchange while exploring this practically, beginning with exchanging ten ones (progressing if children are secure). | Hundreds Tens Ones <br> $0 \varnothing \varnothing \sigma$ 000 $\varnothing 0 \phi \varnothing$ <br>    <br>   $\varnothing \varnothing \varnothing \varnothing \varnothing \varnothing$ <br>    <br> Draw place value counters in a place value chart / be given pictures of place value counters in a place value chart to subtract. Children should be taught when to exchange - relating to their work in the concrete and be able to show this in their drawings. <br> (Showing the abstract alongside the pictorial can help children make the connection). | Start the formal written method by partitioning the numbers into clear place value columns (relating to the concrete and pictorial work). Children should be taught when to exchange and be able to show this in their calculations. <br> Children should progress on to the compact column method to subtract. Children should be taught when to |



| KEY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| vocabulary | digits subtraction take away less fewer difference minus columnn |  |
| method place value column | exchange <br> regroup |  |


| Year 3 - multiplication |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To multiply two-digit numbers by one-digit numbers, using (mental and progressing to) formal written methods. <br> (using timestable facts they are familiar with see below) <br> Year 2 - <br> 2,5,10,3 <br> Year 3 - <br> 4,8,6,11 | $4 \times 13$ (four groups of 13 ) <br> Children use place value counters and/or base ten to show multiplication as repeated addition to multiply a twodigit number by a one-digit number. Children may be beginning to see when an exchange is needed and see that the exchanged value is added NOT multiplied but should find the answer by counting. <br> (Showing the abstract alongside can help children make the connection). <br> Suggested time: 1 lesson with exchanging, 1 lesson without exchanging. | $2 \times 34=$$24 \times 3=72$   <br> $\times$ 20 4 <br> 3 00 0000 <br> 00 0000  <br> 00 0000  <br>  60 12 <br>   60 <br>   $+\frac{10}{72}$ <br> Children then use their own place value counter drawings or drawings that are presented to them to multiply a twodigit number by a one-digit number (either through repeated addition or multiplication). Children may be beginning to see when an exchange is | $18 \times 3=54$ <br> Use the grid as an abstract representation of the concrete and pictorial (arrays). Children may be beginning to see when an exchange is needed and see that the exchanged value is added NOT multiplied but should find the answer by counting. Some children will need pictorial / abstract alongside. <br> Suggested time: <br> Suggested time: 1 lesson with exchanging, 1 lesson without exchanging. |



| KEY |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| VOCABULARY | digits <br> repeated addition | multiplication <br> product | multiply <br> commutative | times array <br> column method | groups of <br> place value column | grid |


| Year 3 - division |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To write and calculate mathematical statements for division <br> (using timestable facts they are familiar with - see below) <br> Year 2 - <br> 2,5,10,3 <br> Year 3 - <br> 4,8,6,11) | Children explore arrays practically and are reminded of the inverse using related multiplication facts to find division facts. E.g. if I know 5 $\times 3=15$, I know $15 \div 5=3$. Children know that dividing means sharing into equal groups. <br> Suggested time: 1 lesson (recap from year 2 although some children may need longer). $84 \div 2=$ | Children draw their own arrays and use line to share into equal groups to find division facts. Some may also need to continue to use circles and drawings to share into equal groups (demonstrate how this method becomes more inefficient as the numbers get larger). | $\begin{aligned} & 5 \times 3=15 \\ & 3 \times 5=15 \\ & 15 \div 5=3 \\ & 15 \div 3=5 \end{aligned}$ <br> Children can find the inverse of known multiplication facts when presented with calculations. <br> Suggested time: Suggested time: 1 lesson (recap from year 2 although some children may need longer). $29 \div 8=3 \text { REMAINDER } 5$ <br> Children are beginning to use known multiplication facts to realise when there is some 'left over' as a remainder. |



|  | Children begin to understand that they can exchange a ten for ten ones to divide further and explore this practically. Children are introduced to having some 'left over' as a remainder. <br> Suggested time: until secure. | Sugges | d time: until secure. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| KEY <br> VOCABULARY | division divide | share | share into equal groups array groups of/lots of | inverse | remainder/left over |


| Year 4 - addition |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To add numbers with up to 4 digits using the formal written methods of columnar addition where appropriatenumbers up to 10,000. | Use place value counters alongside the calculation. Children should practise calculations that include more than one exchange, exchanging tens and hundreds (progressing if children are secure). <br> Suggested time: 1 lesson without exchanging, 2 lessons with exchanging. |  <br> Draw place value counters in a place value chart / be given pictures of place value counters in a place value chart to add. Children should be reminded when to exchange - relating to their work in the concrete and be able to show this in their drawings. <br> (Showing the abstract alongside the pictorial can help children make the connection). | 1378 <br> +2148 <br> 3526 <br> 11 <br> Use the column method to add. Children should be taught when to exchange (linking to their knowledge of the concrete and pictorial) and be able to show this in their calculations. <br> Suggested time: Children should be secure using this method before moving into year 5 (could take a whole week). Some children may need the concrete/pictorial alongside the abstract. |



| Year 4 - subtraction |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To subtract numbers with up to 4 digits using the formal written methods of columnar addition where appropriatenumbers up to 10,000. | Use base ten to subtract alongside the calculation. Children should practise calculations that include more than one exchange, exchanging tens and hundreds (progressing if children are secure). <br> Use place value counters alongside the calculation. Children should practise | Thousands Hundreds Tens Ones  <br>      <br>      <br> Children draw / be given pictures of place value counters in a place value chart and base ten to subtract. Children should be reminded when to exchange - relating to their work in the concrete and be able to show this in their drawings. | $\begin{array}{r} 31 \\ 4357 \\ -2735 \\ \hline 1622 \\ \hline \end{array}$ <br> Use the column method to subtract. Children should be taught when to exchange (linking to their knowledge of the concrete and pictorial) and be able to show this in their calculations. <br> Suggested time: Children should be secure using this method before moving into year 5 (could take a whole week). Some children |



| Year 4 - multiplication |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To multiply two-digit and three-digit numbers by a one-digit number using formal written layout. | $4 \times 126=$ $327 \times 4=$ <br> Use place value counters and/or base ten to show multiplication as repeated addition to multiply a two-digit or a |  <br> Children then use their own place value counter drawings or drawings that are presented to them to multiply a twodigit or three-digit number by a one- | $\times$ 30 5 <br> 7 210 35$210+35=245$ <br> Use the grid as an abstract representation of the concrete and pictorial. Children should be able to see when an exchange is needed when adding the multiplied values. Children need to be secure with multiplication fact e.g. knowing how to calculate $30 \times$ 7. <br> (Showing the column method alongside can help children make the connection). <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging (children should be secure using this method before moving on). |

three-digit number by a one-digit number. Children should be able to see when an exchange is needed and see that the exchanged value is added NOT multiplied.
(Showing the abstract alongside the pictorial can help children make the connection).

Suggested time: 1 lesson without exchanging, 1 lesson with exchanging, beginning with 2-digits progressing to 3-digits.
$36 \times 7=$


This should progress on to using place value counters shown as multiplication
digit number (either through repeated addition or multiplication). Children should be able to show when an exchange is needed and recognise that the exchanged value is added NOT multiplied.
(Showing the abstract alongside the pictorial can help children make the connection).

Suggested time: 1 lesson without exchanging, 1 lesson with exchanging, beginning with 2-digits progressing to 3-digits.


Children use expanded column method to multiply a two-digit or three-digit number by a one-digit number. Children should be able to show when an exchange is needed and know that the exchanged value is added NOT multiplied. Begin by showing the calculations at the side as a link to the grid method.

Suggested time: 1 lesson without exchanging, 1 lesson with exchanging (children should be secure using this method before moving on), beginning with 2-digits progressing to 3-digits


|  |  |  |  | Suggested time: 1 lesson without exchanging, 1 lesson with exchanging (preferred method for year 4/5). |
| :---: | :---: | :---: | :---: | :---: |
| KEY VOCABULARY | digits multiplication repeated addition product | multiply commutative | times array column method | groups of lots of place value column grid |


| Year 4 - division |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To divide two-digit and three-digit numbers by a one-digit number -non-statutory <br> To practise becoming fluent in the formal written method of short division with exact answers -non-statutory | Children use place value counters (alongside the abstract)-sharing hundreds, tens then ones beginning with calculations that don't involve exchanging. Children realise that some can be 'left over' as a remainder. <br> Suggested time: until secure. | Children draw or are presented with pictures of place value counters. Children share into equal groups - sharing hundreds, tens then ones, beginning with calculations that don't involve exchanging. Children realise that some can be 'left over' as a remainder. | Children are introduced to the formal written method of short division alongside the pictorial and abstract. Children should begin with calculation that require one exchange, moving on to calculations that require more than one. <br> Children can show that some division problems have a remainder. <br> Suggested time: 2 lessons, one without exchanging, one with exchanging. |



| Year 5 - addition |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To add whole numbers with more than 4 digits, including using formal written methods (columnar addition) numbers up to $1,000,000$. | Use place value counters along side the calculation. Children should know when to exchange. <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging. |  <br> Draw place value counters in a place value chart to add. Children should know when to exchange and be able to show this in their drawings. <br> (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging. |  Th H T O <br>  3 3 5 6 <br> + 2 4 3 5 <br>  5 7 9 1 <br> Use the column method to add. Children should know when to exchange and be able to show this in their calculations. <br> Suggested time: 1 lesson without exchanging, 1 lesson with one exchange, 1 lesson with multiple exchanges. This should be taught until children are secure - preferred method for upper KS2. |

To practise adding decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1) - nonstatutory decimals with up to 3 decimal places.


Use place value counters alongside the calculation. Children should know when to exchange.

$$
\begin{aligned}
& \text { = } 1 \text { whole } \quad=1 \text { tenth } \\
& \|=1 \text { hundredth }=1 \text { thousandth }
\end{aligned}
$$

Use base 10 (using the representations above) to add decimals. Children will first need to be taught how to represent decimals using base 10. Children should know when to exchange.


Draw place value counters in place value chart to add decimals. Children should know when to exchange and be able to show this in their drawings.
(Showing the abstract alongside the pictorial can help children make the connection).

Use the column method to add decimals. Children should know when to exchange and be able to show this in their calculations.


Children could be taught how to use 0 as a place holder when adding decimals. Possible rule to teach:

|  | (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging. | Use pictures of base 10 to add decimals. Children should know when to exchange. <br> (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging. | Suggested time: 1 lesson without exchanging, 1 lesson with one exchange, 1 lesson with multiple exchanges. This should be taught until children are secure - preferred method for upper KS2. |
| :---: | :---: | :---: | :---: |
| KEY <br> VOCABULARY | digits addition add place value column exch | more sum altogether regroup decimal commutative | total columnn method decimal places |


| Year 5 - subtraction |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Objective/s | Concrete |  |  |  | Pictorial |  |  |  | Abstract |  |  |  |
| To subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction) numbers up to 1,000,000. |  | place value calculation when to <br> gested tim hanging, 1 hanging. | $\qquad$ <br> 3 <br> 6 <br> counter Children exchange <br> e: 1 lesso esson with |  | Draw place value counters in a place value chart to subtract. Children should know when to exchange and be able to show this in their drawings. <br> (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging. |  |  |  |  Th H T O <br>  5 6 3/ $1_{3}$ <br> - 4 3 1 6 <br>  1 3 2 7 <br> Use the column method to add. Children should know when to exchange and be able to show this in their calculations. <br> Suggested time: 1 lesson without exchanging, 1 lesson with one exchange, 1 lesson with multiple exchanges. This should be taught until children are secure - preferred method for upper KS2. |  |  |  |

```
To practise
subtracting
decimals,
including a mix
of whole
numbers and
decimals,
decimals with
different
numbers of
decimal places,
and
complements of
1 (for example,
1-0.83=
0.17) - non-
statutory -
decimals with
up to 3 decimal
places.
```



Use place value counters alongside the calculation. Children should know when to exchange.

$0.64-0.33=$ ?

Use base 10 to subtract decimals. Children will first need to be


Draw place value counters in place value chart to subtract decimals. Children should know when to exchange and be able to show this in their drawings.
(Showing the abstract alongside the pictorial can help children make the connection).

$$
\begin{array}{r}
{ }^{4} 5^{1} .43 \\
-2.7 \\
\hline 2.73 \\
\hline
\end{array}
$$

Use the column method to subtract decimals. Children should know when to exchange and be able to show this in their calculations.


Children could be taught how to use 0 as a place holder when subtracting decimals.

Suggested time: 1 lesson without exchanging, 1 lesson with one exchange, 1 lesson with multiple exchanges. This


| Year 5 - multiplication |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To multiply numbers up to 4 digits by a one- or twodigit number using a formal written method, including long multiplication for two-digit numbers | Use place value counters as repeated addition to multiply a four-digit number by a one-digit number. Children should be able to see when an exchange is needed and see that the exchanged value is added NOT multiplied. <br> (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson. | Children then use their own drawings or drawings that are presented to them to multiply a four-digit number by a one-digit number. Children should be able to show when an exchange is needed and recognise that the exchanged value is added NOT multiplied. <br> (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson. |  Th H T O <br>  1 8 2 6 <br> $\times$    3 <br>  5 4 7 8 <br>  2 1   <br> Children use the column method to multiply a four-digit number by a onedigit number. Children should be able to show when an exchange is needed and know that the exchanged value is added NOT multiplied. <br> Suggested time: Children need to be secure with this method before moving on to multiplying 2-digits by 2-digits preferred method for upper KS2. |




| (Showing the abstract alongside the pictorial can help children make the connection - showing the grid will help make the clearest connection here). <br> Suggested time: 1 lesson for threedigits, 1 lesson for four-digits. |  | This should progress on to using the compact column method (the calculations at the side as a link to the grid - once secure, these can be left out). <br> Suggested time: $2 / 3$ lessons - children should be secure using this method before moving in to year 6 and before moving on to multiplying a three-digit or four-digit number by a two-digit number. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\times$ | 200 | 30 | 4 |
|  |  | 30 | 6,000 | 900 | 120 |
|  |  | 2 | 400 | 60 | 8 |





| KEY |
| :---: | :---: | :---: |
| VOCABULARY |$\quad$ division divide share share into equal groups remainder/left over | groups of/lots of |
| :---: |




Use place value counters alongside the calculation. Children should know when to exchange.

$$
\begin{aligned}
& \text { = } 1 \text { whole } \quad=1 \text { tenth } \\
& \|=1 \text { hundredth }=1 \text { thousandth }
\end{aligned}
$$

Use base 10 (using the representations above) to add decimals. Children will first need to be taught how to represent decimals using base 10. Children should know when to exchange.

(1)


Draw place value counters in place value chart to add decimals. Children should know when to exchange and be able to show this in their drawings.
(Showing the abstract alongside the pictorial can help children make the connection).

### 3.65 <br> $+2.41$ <br> 6.06 <br> 1

Use the column method to add decimals. Children should know when to exchange and be able to show this in their calculations.


Children could be taught how to use 0 as a place holder when adding decimals. Possible rule to teach:

|  | (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging. | Use pictures of base 10 to add decimals. Children should know when to exchange. <br> (Showing the abstract alongside the pictorial can help children make the connection). <br> Suggested time: 1 lesson without exchanging, 1 lesson with exchanging. | Suggested time: 1 lesson without exchanging, 1 lesson with one exchange, 1 lesson with multiple exchanges. This should be taught until children are secure - preferred method for upper KS2. |
| :---: | :---: | :---: | :---: |
| KEY <br> VOCABULARY | digits addition add place value column exch | more sum altogether regroup decimal commutative | total columnn method decimal places |


| Year 6 - subtraction |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Objective/s | Concrete |  |  |  | Pictorial |  |  |  | Abstract |  |  |  |  |
| To practise subtraction with larger numbers, using the formal written methods (columnar subtraction) numbers up to 10,000,000. |  | place val calculatio when to <br> gested ti anging, anging. |  | alongside should <br> without | Draw plac value char should kn able to sh <br> (Showing pictorial connection) <br> Suggeste exchangin | 5643 100 s -8 88 <br> value <br> to sub <br> when <br> w this <br> he abs <br> $n$ help <br> time: <br> 1 less | 4316 <br> 10s <br> -( <br> unters <br> act. C <br> exch <br> their <br> ct alo <br> ildren <br> sson <br> with | in a place idren nge and be rawings. <br> gside the make the <br> thout xchanging. | Use should to sh <br> Sugg exch lesso shou - pre |  | H <br> 6 <br> 3 |  | 0 <br> 13 <br> 6 <br> 7 <br> add. Children ange and be able lations. <br> without one exchange, 1 nges. This dren are secure per KS2. |


| To practise |
| :--- |
| calculations |
| using decimal |
| fraction |
| equivalents to |
| aid fluency. |



Use place value counters alongside the calculation. Children should know when to exchange.

$0.64-0.33=$ ?

Use base 10 to subtract decimals. Children will first need to be


Draw place value counters in place value chart to subtract decimals.
Children should know when to exchange and be able to show this in their drawings.
(Showing the abstract alongside the pictorial can help children make the connection).

$$
\begin{array}{r}
{ }^{4} 5^{1} .43 \\
-2.7 \\
\hline 2.73 \\
\hline
\end{array}
$$

Use the column method to subtract decimals. Children should know when to exchange and be able to show this in their calculations.


Children could be taught how to use 0 as a place holder when subtracting decimals.

Suggested time: 1 lesson without exchanging, 1 lesson with one exchange, 1 lesson with multiple exchanges. This


| Year 6 - multiplication |  |  |
| :---: | :---: | :---: |
| Objective/s | Concrete | Abstract |
| To multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication |  | $\times$ 200 30 4 <br> 30 6,000 900 120 <br> 2 400 60 8 <br> Use the grid as an abstract representation of the concrete and pictorial. Children should be able to see when an exchange is needed when adding the multiplication answers. Children need to be secure with multiplication fact e.g. knowing how to calculate $20 \times 30$. <br> (Showing the column method alongside this can help children make the connection). <br> Suggested time: if needed. |



| Multiply onedigit numbers with up to 2 decimal places by whole numbers |  |  | If secure, children could be shown how to use the column method to multiply (this is non-statutory and should only be taught if children are secure with the concrete and pictorial). |
| :---: | :---: | :---: | :---: |
| KEY VOCABULARY | digits multiplication <br> repeated addition product <br> decimal place  | multiply times arraycommutativedecimal numbercolumn method <br> decimal poin | groups of lots of place value column grid integer |


| Year 6 - division |  |  |  |
| :---: | :---: | :---: | :---: |
| Objective/s | Concrete | Pictorial | Abstract |
| To divide numbers up to 4 digits by a two-digit number using the formal written method of short division. <br> To divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division |  | See year 5 for skills recap. | $496 \div 11 \text { becomes }$ <br> Answer: $45 \frac{1}{11}$ <br> Children are re-introduced to the formal written method of short division but this time dividing by a 2 -digit number (where appropriate e.g. times tables facts they know or can easily work out). This should begin with calculations that require 0-1 exchanges, moving on to calculations that require more than one. This should include problems that have remainders (some problems may require the remainder to be presented in fraction / decimal form). <br> Suggested time: until secure. |



|  |  |  |  | Suggested time: until secure. |
| :---: | :---: | :---: | :---: | :---: |
| KEY <br> VOCABULARY | division divide shareshare into equal groups <br> groups of/lots of | remainder/left over |  |  |

